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SURVEY ON MOVING ICE

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SURVEY ON MOVING ICE

A Sixty Kilometres Long Triangulation Net across an Antarctic Ice Shelf

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Abstract

In order to measure ice deformation at frequent intervals across the Brunt Ice Shelf, Antarctica, 64 markers were planted to form a network of overlapping braced quadrilaterals. This network was triangulated in 1966 and, after a 10-month interval, again in 1967. Throughout the periods of observation the survey figures suffered continual distortion, and any computation method must allow for this. Making the assumption that ice movement at each marker is linear it is possible to interpolate the positions of observed stations and to process the triangulation scheme as a series of resections. To solve these a computer-oriented resection solution was developed, and adapted to accept measured lengths in addition to angles as input data. In order to correct for Earth-curvature, and to utilise observed true azimuths, a simple spheroid model was adopted.

The survey yielded positions of all markers at the times of occupation; position accuracy relative to the origin of the scheme was 1 : 40,000. Values of ice velocity deduced from these positions ranged from 50 metres per year on the grounded inland ice sheet to 350 m yr^{-1} at the front of the floating ice shelf.

1 Introduction

Since the International Geophysical Year of 1957–1958, the polar regions have attracted the attention of an ever-increasing number of scientists. This growing interest has exposed several problems which are special to the area; one of these is the problem of applying surveying techniques to the surface of moving ice. To gain information on ice deformation, to supply control for mapping purposes, or to transfer control over long distances it is often necessary to fix positions on the ice itself.

Before conventional surveying techniques can be applied, some account must be taken of the continual relative movement of the stations comprising the survey scheme. All lengths and angles must be observed on at least two occasions and the data then processed by a "reduction to epoch" method, assuming:

(i) Rate of change of each observed quantity is constant (Swithinbank, 1957, p. 86); reduction to epoch is achieved by linear interpolation of the observed values; or

(ii) The ice velocity at any point is constant. If both angles and lengths are observed, they can be reduced to epoch values in terms of their values at the times of observation; if not, the positions of survey stations can be reduced to epoch values.

These methods have been rigorously applied to a traverse across the Ross Ice Shelf where lengths and angles between adjacent members of a 910 km single line of stakes were measured (Dorrer, 1969). The field work occupied six men for two periods, each of two months, and resulted in a measure of ice velocity at each of 103 markers.

Principal strain rates at the surface could be computed only where the stake line took a markedly zig-zag course, and for a more complete investigation of relative ice deformation it is necessary to measure the velocities of an array of stakes; principal strain rates within any given stake triangle can then be calculated. Logistically, the simplest method of obtaining this information is by triangulation, which requires only one party of two men to complete the observations. Reduction to epoch of stake positions enables the scheme to be computed as a series of resections.

This approach has been applied to the measurement of ice velocity and strain patterns on the Brunt Ice Shelf in Antarctica, and the survey methods, computation and results are briefly described here.

2. Field Work

2.1 *Terrain*

The Brunt Ice Shelf is a 200 m thick floating ice sheet fringing the east coast of the Weddell Sea. A British Antarctic Survey station on the ice shelf is situated at 75° 30·5′ S., 26° 30′ W., 3·5 km from the ice front and 54 km north-west of the junction of ice shelf and inland ice sheet (Fig. 1); the nearest rock outcrops are 360 km away. The ice front consists of cliffs 5 m to 30 m high intersected by occasional narrow inlets. The surface of the ice shelf rises slowly to a maximum of perhaps 50 m above sea-level near areas of grounded ice and, apart from gentle undulations of about 1 km wavelength, and up to 5 m waveheight, it is almost flat; consequently, Earth curvature prevents visibility over distances much greater than 8 km.

Some 12 km east of Halley Bay station the ice shelf is intensely disturbed where it locally runs aground. This disturbance takes the form of sub-parallel pressure ridges, the McDonald Ice Rumples, up-stream from the grounding, and intense crevassing near and over the grounded ice itself.

Near its southern margin with the grounded inland ice sheet the ice shelf becomes more undulating, the ridges finally becoming recognisable as drift-modified fragments of "icebergs" calved from the inland ice sheet (Swithinbank 1957, p. 26; Barclay 1964, p. 334). The grounded ice of the inland ice sheet rises by steep, severely crevassed slopes from the ice shelf, the "hinge-line" between the two being impassable along most of its length. Whilst the disturbed terrain near the inland ice sheet and the McDonald Ice Rumples hinders travel, the high ridges associated with such areas allow for better visibility between stations.

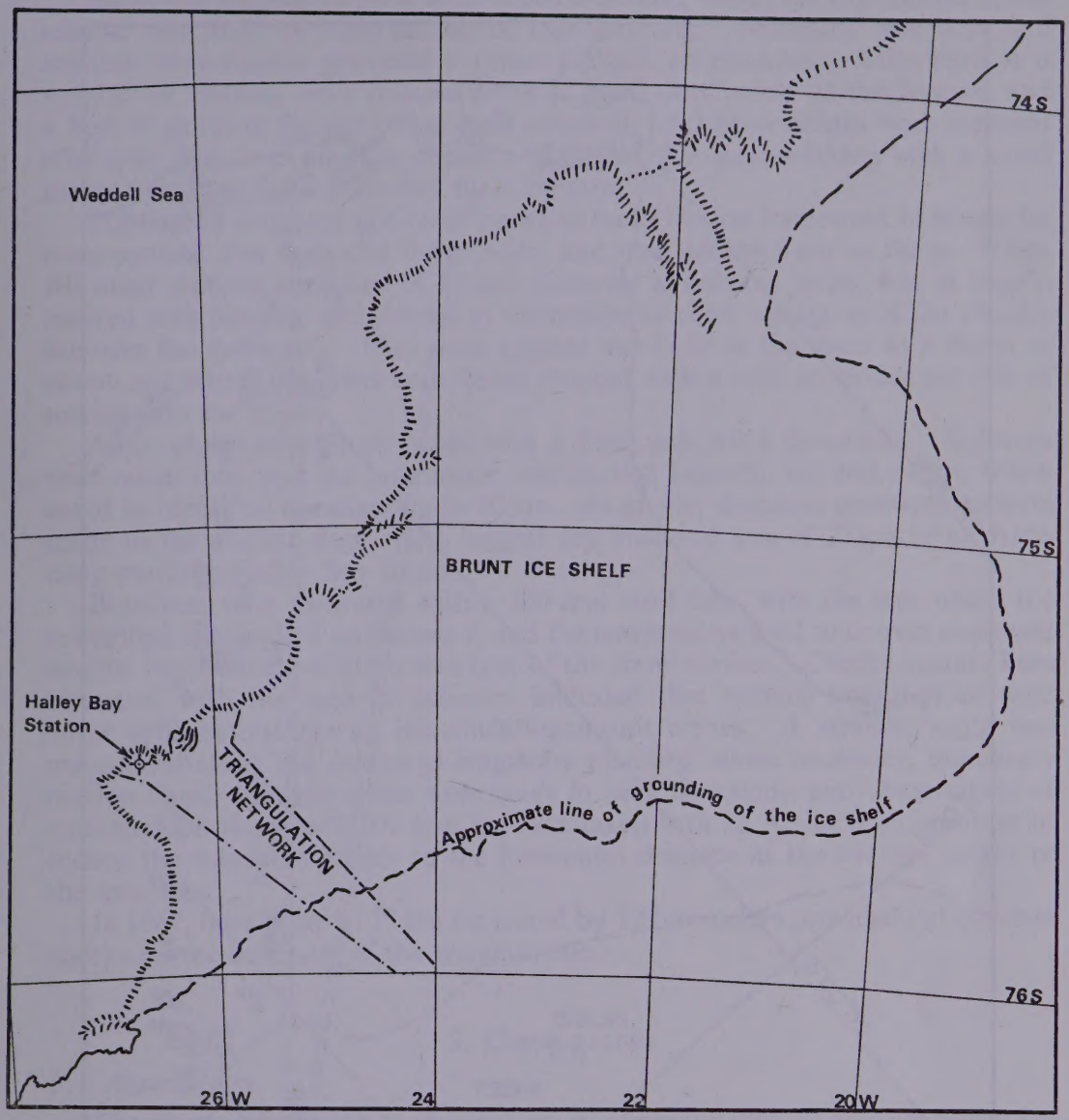

Fig. 1. Sketch Map of the Brunt Ice Shelf showing area studied.

2.2 *Method*

In order to describe ice particle trajectories between the inland ice sheet and the ice front, measurements were made of surface ice velocities at frequent intervals along a flow-line.

The method adopted was a triangulation scheme consisting of a series of linked, braced quadrilaterals of side length 3–5 km, extending from the origin (the beacon) on the grounded ice of the McDonald Ice Rumples, across the ice shelf and on to the inland ice sheet 60 km to the south-east (Fig. 2). At its northern end the network of stations was extended laterally in order to provide additional information about the behaviour of the ice in the vicinity of the McDonald Ice Rumples.

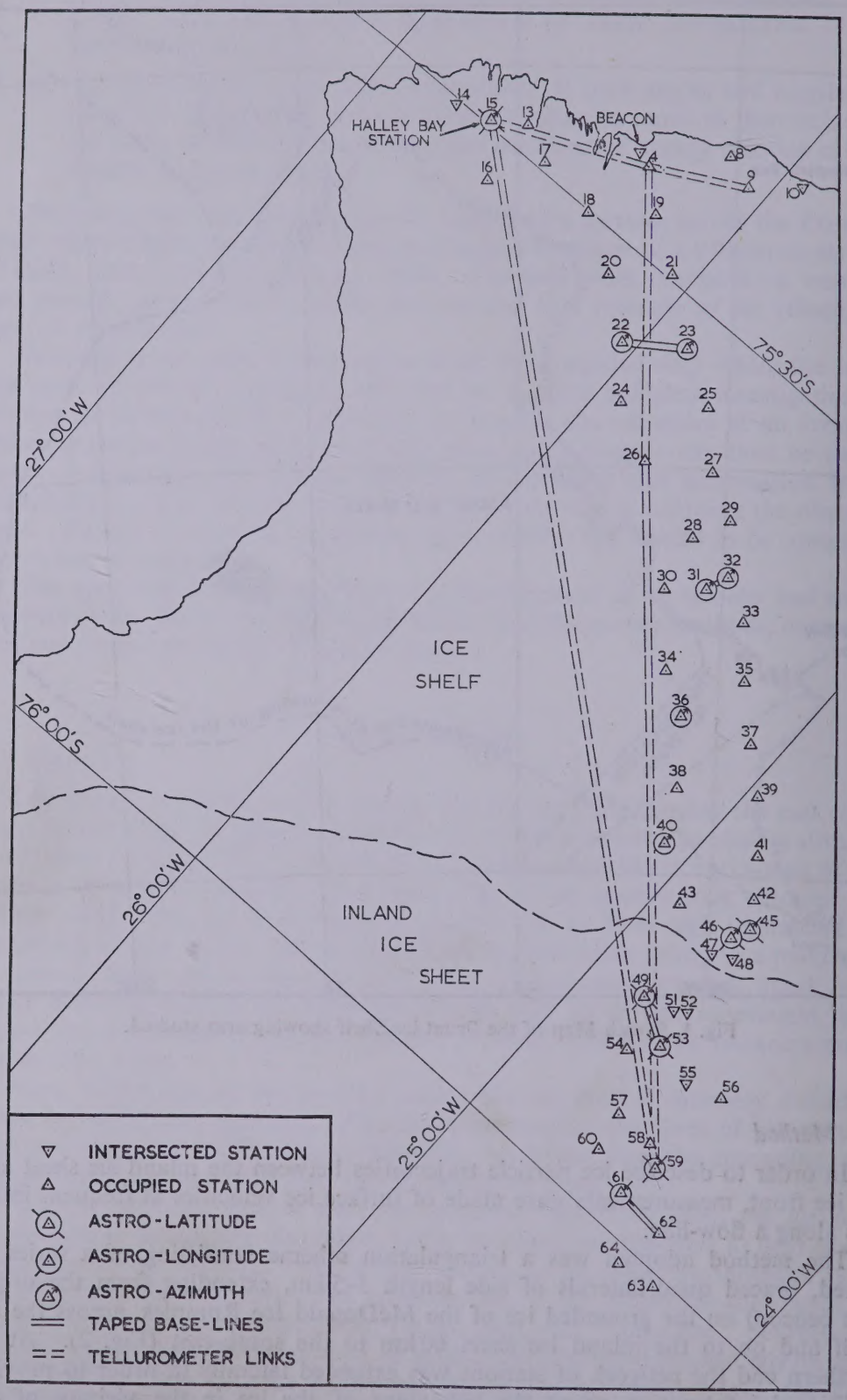

Fig. 2. Part of the Brunt Ice Shelf showing positions of the stakes comprising the triangualtion

Scale was introduced by a 5 km taped base-line, whilst the orientation of the scheme was fixed by observations of true azimuth. Additional base-lines and azimuth observations provided a running check on accuracy. Astro-fixes at a number of stations were reduced to their equivalent values at the beacon, and a best fit position for the origin thus obtained. All observations were repeated after one year, and on each occasion occupied two men working with a small motor toboggan for a period of three months.

The beacon consisted of a black plywood faced Dexion framework of triangular cross-section, 3 m high and 0·4 m wide, and was sighted from as far as 30 km. All other stations consisted of 25 mm diameter aluminium poles, 4 m in length, marked with bunting, and jointed in the middle to allow setting up of the theodolite over the stake site. They were planted vertically in the snow to a depth of about one metre, the lower ends being plugged with a cork to reduce the risk of sinking into the snow.

Angle observations were made with a Tavistock Mk I theodolite. Sightings were taken into, and the instrument was centred beneath, the stake tops, which could be identified at ranges up to 10 km. At greater distances observations were made to the marker flags. The longest ray observed was of 77 km to an Aldis lamp marking Halley Bay station.

Base-lines were measured with a 300-foot steel tape, with the tape under the prescribed tension laid on the snow, and the temperature used to correct measured lengths for thermal variation was that of the snow surface. Checks against lines measured with the tape in catenary indicated that surface irregularities were never sufficiently large to introduce significant errors. A straight route was maintained along the measured length by planting, where necessary, temporary marker flags. Measurements were made in both directions, and observations of surface slope along each 300 foot leg were taken with an Abney level, in order to reduce the measured length to the horizontal distance at the average height of the base-line.

In 1967, lines of up to 77 km measured by Tellurometers provided independent checks on the accuracy of the triangulation.

3. Computation

3.1 *Assumptions*

The simplest approach to the problem of surveying over an actively deforming surface is to reduce all observations to their equivalent values on two epoch dates on the assumption that the rates of change of angle and distance from any station are constant (Swithinbank, 1957). The problem then reduces to one of solving two essentially independent schemes, which can be processed by a conventional least-squares method, computer programs for which have been perfected and are in regular use. However, before embarking on this course, the validity of the assumptions was checked. To this end, observations at a number of stations had been made on three or more occasions, and the of rates change of angle (*Ra*) over the periods between observations for a selection of these stakes have been used to compare observed angles on the intermediate observation date with those obtained by interpolation assuming *Ra* to be constant (Table 1). *Ra* is effectively constant for stakes 22, 23, 26, 29, 32, and 57, situated in the region of relatively uniformly strained ice (Fig. 2), but for stakes 4, 8, 9, 18, 19 in the vicinity of the McDonald Ice Rumples, *Ra* is changing significantly

TABLE 1

COMPARISON OF EPOCH-REDUCED ANGLES WITH OBSERVED ANGLES

Stake No.	Mean residual (observed angle—epoch-reduced angle at intermediate observation date) *seconds of arc*	No. of days separation of intermediate observation from initial or terminal date, whichever is nearer.	Approx. error resulting from 20 days separation. *seconds of arc*
4	13	1·8	150
8	9	6	30
9	28	33	17
18	12	4·1	59
19	58	37	31
29	13	225	1
32	5	228	0
23	4	220	0
22	3	236	0
26	3	244	0
57	10	195	1

with time. Because of the time taken to complete observations of figures linking the ice shelf and inland ice sheet, a separation between observation and epoch date of approximately 20 days in places introduced an error of more than 30 seconds of arc to the epoch reduced angles.

Consequently, a more rigorous approach was attempted, and the only assumption made was that over the total period of the survey (13 months) the rate and direction of the ice movement at any stake site remained constant. By using for control points the time-reduced positions of observed stakes, the triangulation scheme can be considered as a series of resections.

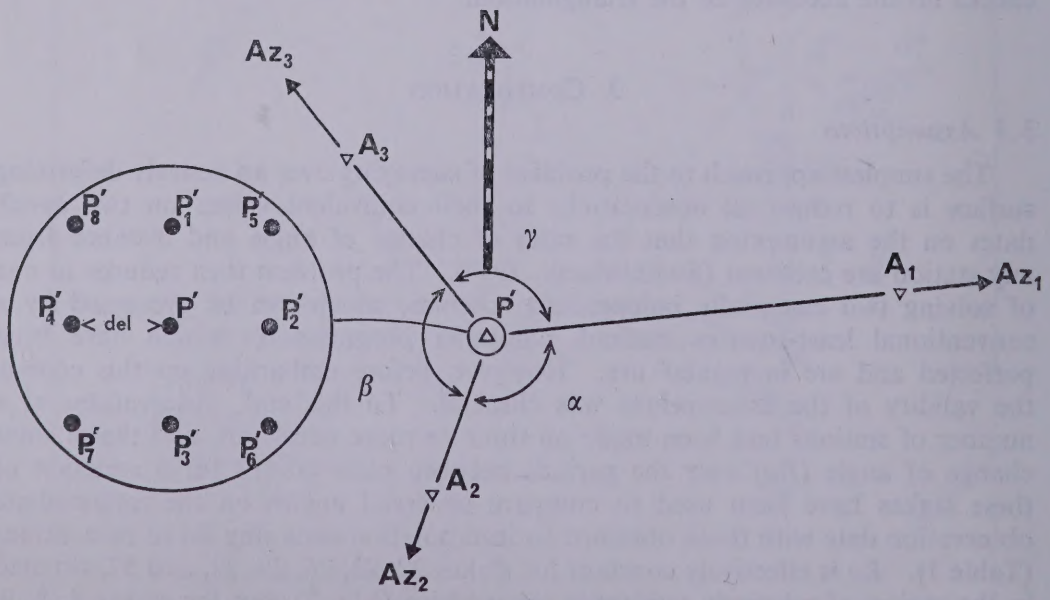

Fig. 3. A three-point resection. The inset illustrates the pattern of trial positions tested by the "shuffle resection" solution.

3.2 *Shuffle resection*

The method used to solve the resections should be sufficiently flexible to accept any number of control points and to give an indication of the accuracy achieved. By minimising the sum of the squares of the residuals (observed—computed) of observations from the unknown stake, a best fit position can be found. Fig. 3 shows a simple resection of an unknown position P, using control points A_j ($j = 1-3$); the computation follows these stages:

(a) True azimuth Az_j is computed for rays from an assumed position P' into control points A_j. The sum of the squares of the residuals then becomes:

$$S = [(Az_2 - Az_1) - \alpha]^2 + [(Az_3 - Az_2) - \beta]^2 + [(Az_1 - Az_3) - \gamma]^2,$$

(b) (a) is repeated for all P_i' ($i = 1$–4), and the position giving a minimum S is designated as P′; the whole process is repeated until no further reduction in S occurs.

(c) (a) is repeated for all P_i' ($i = 5$–8). If this gives smaller S, then (b) is repeated; if not *del* (see Fig. 3) is reduced and the whole process is repeated.

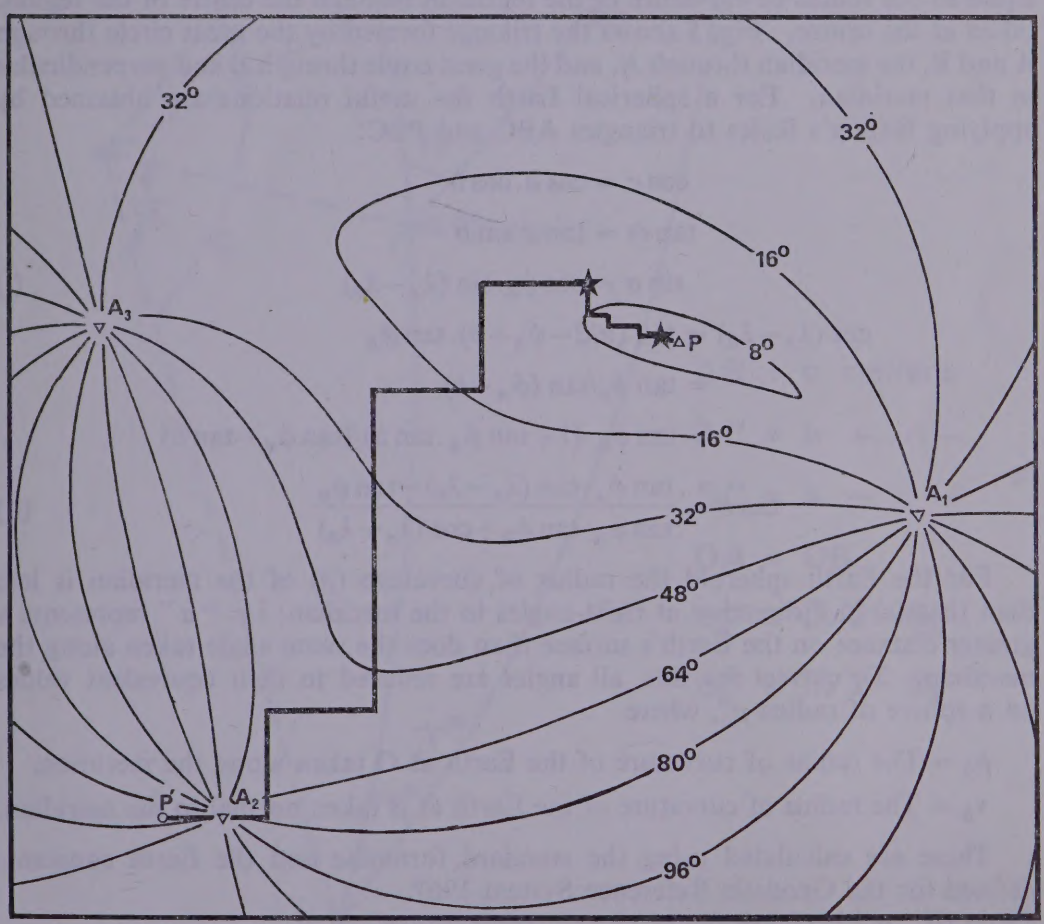

Fig. 4. Contour Map of the r.m.s. residual surface to the resection problem illustrated in Fig. 3 The values of the r.m.s. residuals (observed—computed angle) are printed alongside the contours. The thick line describes the route taken by the sampled postions P′ in finding the best fit position P.

(d) When *del* is sufficiently small the P′ resulting from (c) is accepted as the "best fit" position P.

Typical limits for *del* are: 3000 metres reducing by factors of ten to 0·03 metres, so the originally input assumed position can be readily scaled from a sketch map.

In Fig. 4 the same computation is represented diagrammatically. The r.m.s. residual surface is contoured and shows the minimum P to which the stake position, originally assumed to be at P′, has moved. The stars mark the points at which the shuffle unit *del* has been decreased to a tenth of its former value.

The method can easily accept any number of control points, and can be extended to test observed lengths and true azimuths.

3.3 *Spheroid surface*

In order to operate this method on the Earth surface, thus allowing the use of true azimuth observations, and avoiding "spherical excess" corrections, it was necessary to adopt a spheroid model which combined the required accuracy with simplicity. This was done by adjusting the local geodetic parameters so that the region of immediate interest could be considered as a spherical surface with radius equal to the radius of curvature of the meridian through the centre of the region, taken at the centre. Fig. 5 shows the triangle formed by the great circle through A and B, the meridian through A, and the great circle through B and perpendicular to that meridian. For a spherical Earth the useful relations are obtained by applying Napier's Rules to triangles ABC and PBC:

$$\cos c = \cos a.\cos b$$

$$\tan \omega = \tan a/\sin b$$

$$\sin a = \cos \phi_B.\sin (\lambda_A - \lambda_B) \qquad \text{(i)}$$

$$\begin{aligned}\cos (\lambda_A - \lambda_B) &= \tan (\pi/2 - \phi_A + b).\tan \phi_B \\ &= \tan \phi_B/\tan (\phi_A - b) \\ &= \tan \phi_B.(1 + \tan \phi_A.\tan b)/(\tan \phi_A - \tan b)\end{aligned}$$

or

$$\tan b = \frac{\tan \phi_A.\cos (\lambda_A - \lambda_B) - \tan \phi_B}{\tan \phi_A.\tan \phi_B + \cos (\lambda_A - \lambda_B)} \qquad \text{(ii)}$$

For the Earth spheroid the radius of curvature (ρ) of the meridian is less than that (v) in a direction at right-angles to the meridian; i.e. "a" represents a greater distance on the Earth's surface than does the same angle taken along the meridian. To correct for this, all angles are reduced to their equivalent values on a sphere of radius ρ_0, where

ρ_0 = The radius of curvature of the Earth at O taken along the meridian,

v_B = The radius of curvature of the Earth at B taken normal to the meridian.

These are calculated using the standard formulae and the Earth constants defined for the Geodetic Reference System 1967.

From (i) "a" becomes

$$(v_B/\rho_0).\arcsin (\cos \phi_B.\sin (\lambda_A - \lambda_B))$$

i.e.

$$\sin a = \cos \phi_B.\sin \text{DIFLON}$$

where

$$\text{DIFLON} = \arcsin\left(\frac{\sin\left[(v_B/\rho_0).\arcsin\left(\cos\phi_B.\sin\left(\lambda_A-\lambda_B\right)\right)\right]}{\cos\phi_B}\right)$$

By replacing $[(\lambda_A-\lambda_B)]$ in (i) and (ii), by DIFLON, w and c can be calculated for any two points, A and B, whose geographic co-ordinates are known.

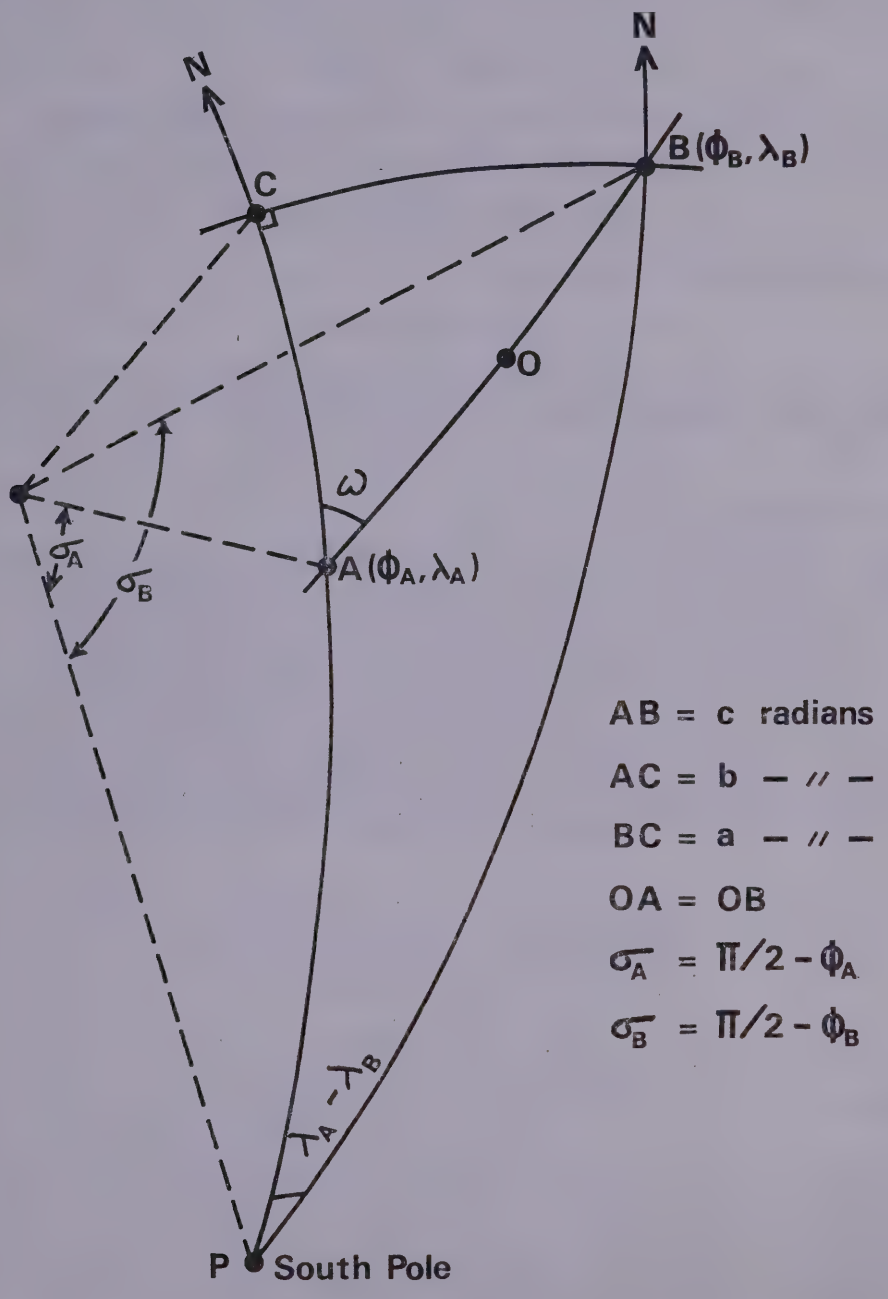

Fig. 5. Spheroid Model.

Azimuths and distances obtained by this method for points 1–100 km apart were checked against corresponding values obtained by precise computation (Rainsford, 1949, pp. 28, 29). Azimuths agreed to within one second of arc, and distances greater than 5 km to within 10 cm. The error in distance estimate (D) increased with decreasing separation of points; this is because D is obtained from the cosine of the angle (c) subtended at the centre of curvature, and the accuracy to which this can be represented depends on the number of significant figures used. The Cambridge University Atlas computer uses 11 for which the smallest change in cosine detectable $(\mathrm{d}(\cos c))$ is 10^{-11}.

$$\mathrm{d}(\cos c)/\mathrm{d}c = -\sin c.$$

Since $\sin c$ is approximately equal to c, which equals D/(Re), where

$$\mathrm{Re\ (Earth's\ radius)} \simeq 6{\cdot}4 \times 10^6 \text{ metres},$$

$$\mathrm{dD} = \mathrm{d}c.\mathrm{Re} = \mathrm{d}(\cos c) \times \mathrm{Re}^2/\mathrm{D}.$$

And for a base-line of 5000 metres

$$\mathrm{dD} = 0{\cdot}08 \text{ metres}.$$

Thus a scale error of 1 : 60,000 is introduced during computation. This is approximately equal to that estimated for measured base-lines.

3.4 *Resection using moving control points*

All survey stations on the ice are moving at different velocities throughout the period of the survey, and in order to apply a method of resection to these stations the positions of control points at the time they were observed must be used. These are obtained by linear interpolation of the geographic co-ordinates available from already completed resections. Because the stake displacements are never more than 400 m, the error introduced to the interpolated positions by Earth curvature is negligible. Positions were computed using only observations into already fixed points, whilst all the observations were used to check the progress and accuracy of the computation. The " shuffle " resection method was adapted

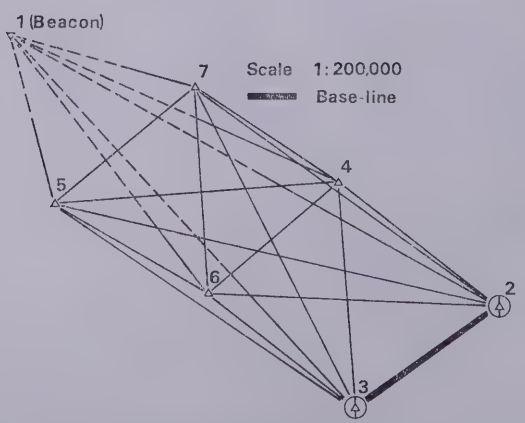

Fig. 6. The first seven stakes to be fixed. This pattern of interlocking braced quadrilaterals continued throughout the network of stakes.

to accept observations of lengths and true azimuths in addition to angles as input data, and was programmed for the Cambridge Atlas computer. The program accepted all available reduced adjusted observations, and output "best fit" stake positions appropriate to the times of observation complete with residuals to all input data.

Fig. 6 shows the first seven stakes fixed. The positions of at least three must be known before any others can be fixed by resection.

(a) A position obtained from astro-fixes was assigned to the beacon which was assumed to be stationary.

(b) The length 1–2 was calculated from each year's measurements of the base-line 2–3 and the angles $\overset{\frown}{1\ 2\ 3}$ and $\overset{\frown}{1\ 3\ 2}$, all reduced to the times of observation from stake 2 assuming constant rate of change. This, together with observations of true azimuth of the line 2–1, was sufficient to fix the positions occupied by stake 2 at the times of observation.

(c) The measured base-line 2–3, the true azimuth 3–2, and the observed angle $\overset{\frown}{1\ 3\ 2}$ were used to give the positions of stake 3.

(d) The remaining stakes were fixed by resection.

When all positions had been computed the input observations were checked against corresponding computed values, and the residuals were printed out together with the stake positions at the times of observation.

Because the length 1–2 was found by solving an incompletely observed triangle using angles which were obtained in an area subjected to intense local strain, it suffers an indeterminate error. To reduce the effects of this, the computation (a–d) was repeated, using values for 1–2 which slowly varied about the computed value, until a minimum r.m.s. residual for the whole net of seven stakes was found. The positions given by this run then supplied initial control for the computation of the remainder of the stake positions. In order to economise on computer storage space the 66 stations comprising the whole net were processed in three batches of approximately 25 stakes each, with sufficient overlap to ensure that all computed stakes visible from the batch about to be processed were included as initial control.

For those stakes which had not been occupied, it was usually possible to reduce to epoch the angles observed into the stake, calculate the intersected angles, and obtain a position by resection using the computer program. It was necessary to fix only six stakes by calculation of the side lengths of incompletely observed triangles.

If, during computation, the residuals at any stake exceeded a pre-determined maximum, the program terminated and a warning was printed giving the relevant station number. Examination of the data for that stake usually revealed target misidentification or garbling of the data.

Although the astro-fix errors apply to the co-ordinates assigned to the beacon, the internal accuracy of the survey is virtually unaltered. All positions used in true azimuth determinations were obtained from the triangulation results, and each suffered an error equal to that of the beacon's co-ordinates. This had the effect of rotating the stake array about the beacon by a small angle which was approximately the same for each survey, so the calculated ice velocities are unaffected.

4. Accuracy

Observation errors are summarised in Table 2.

Table 2

Errors

Cause of error	Approximate error	Equivalent error in angle observed over 4 km ray. *seconds of arc*
Error in observed angles:		
(i) Random errors represented by mean standard errors of the observed angles. Apart from observer and instrument errors, they stem mainly from vibration and shimmer of observed stakes.		
(a) Inland ice sheet, Oct. 1966		±1·72
(b) Ice shelf, Nov./Dec. 1966		±1·24
(c) Inland ice sheet, Oct. 1967		±1·58 } 1·6
(d) Ice shelf, Nov./Dec. 1967		±1·61
(ii) Leaning of targets in a strong wind	4 cm	±2
(iii) Misplacement of theodolite	2 cm	±1
(iv) Horizontal refraction—probably negligible except when sighting across slopes on the inland ice sheet		
Errors in base-lines:		
(i) Random errors exposed in the observed misclosure	1 : 150,000	
(ii) Systematic errors in estimation of tape temperature and surface slope, or failure to maintain a straight measuring route, increase the error to approximately:	1 : 50,000	±4
*Errors in astronomical observations:**		
The standard errors of several results at any one station:		
(a) Latitude by circum-meridional observations of the Sun	±2″ lat.	
(b) Longitude by balanced pairs of observations of the Sun near prime vertical	±5″ long.	
True azimuth by balanced pairs of observations of the hour angle of the Sun	±4″ arc	
Tellurometer errors:		
Instrument errors	Negligible	
Refraction errors† in 50–80 km rays	±1 : 50,000	

* *Because the astro observations extended over a number of days, and the chronometer was frequently checked throughout the period, the error estimates include both chronometer and observing errors. However, they include neither latitude errors due to incorrect estimates of refraction correction, nor errors due to deviation of the vertical.*

† *Inadequate information concerning the temperature and humidity of the air between the end points of long tellurometer measured rays causes very large increases in the errors* (*Saastamoinen*, 1968).

4.1 *Propagation of errors through the scheme*

Consider one limb of the stake line; with respect to its neighbours each station X lies within a circle of uncertainty of radius R_x, where all R_x are interdependent. If we assume R_x is constant for all X, and equal to R, then the error ΔD in the distance D separating any two stakes is:

$$\Delta D = N.R \qquad N = \text{No. of intermediate legs.}$$

or $$\Delta D = D.R/A \qquad A = \text{Mean distance between adjacent stakes} \simeq 4\,\text{km}$$

where R/A = the mean effective error, in radians, of angles observed.

$$= (1{\cdot}6^2+2^2+1^2+4^2)^{\frac{1}{2}}.\sin 1'' \text{ from Table 2,}$$

and $$R \simeq \pm 10\,\text{cm}$$

The error in distance separating stakes on the inland ice sheet from the beacon ($D \simeq 70$ km)

$$= \pm 1{\cdot}7\,\text{m}$$

which is comparable with errors involved in tellurometer measurements over the same distance.

Errors in the estimation of true azimuth used to orientate the scheme result in a random rotation of the whole net by ± 4 seconds of arc each year (Table 2). This leads to an additional position error in the direction of rotation equal to

$$\pm(4D.\sin 1'').$$

So the position error E_x of any stake X is:

$$E_x = D_x.(1{\cdot}6^2+2^2+1^2+4^2+4^2)^{\frac{1}{2}}.\sin 1'',$$

where D_x = distance separating X from the beacon.
And stake velocities V_x derived from comparison of each year's stake positions are in error by:

and $\left.\begin{array}{l}\pm\sqrt{2}.E_x \text{ metres per year} \\ \pm\sqrt{2}.E_x/V_x \text{ radians in direction}\end{array}\right\}$ reaching a max. of $\pm 3{\cdot}5\,\text{m yr}^{-1}$ and $\pm 3°$ of arc on the inland ice sheet

The errors in relative velocities of two stakes separated by D is $\pm\sqrt{2}.D.R/A$, so strain rates are in error by:

$$\pm\sqrt{2}.R/A = \pm 3{\cdot}3\times 10^{-5}\,\text{yr}^{-1}.$$

The residuals to all observed quantities are output by the computer and can be compared with the errors estimated above:

Mean effective S.E. of observed angles (Table 2)	$= \pm(1^2+2^2+1{\cdot}6^2)^{\frac{1}{2}}$ $\simeq \pm 3''$
R.M.S. residual (observed–computed) to angles	$= \pm 2''$ on the ice shelf $\pm 4''$ on the inland ice sheet
Error estimated above for the distance separating the beacon from the stakes on the inland ice sheet	$= \pm 1{\cdot}7\,\text{m}$

Mean residual (Tellurometer observation–computed length) for three measurements from ice shelf to inland ice sheet $= \pm 1{\cdot}3$ m

5. Results

This work yielded positions of all stakes at the times of occupation, or an assigned epoch in the case of intersected stakes. These positions were input to a second program which reduced the positions to those occupied on two selected dates a year apart, and calculated stake velocities and principal strain rates and directions for ice within any selected triangle of stakes. A third program produced a plotter output illustrating all the information gained.

Table 3

Comparison of Independent Estimates of Ice Velocity at Halley Bay Station

Source (see text)	Interval	Mean velocity and direction	
		m yr^{-1}	east of true north
(i)	1959	366 ± 40	$266° \pm 5°$
(ii)	1957–1967	396 ± 25	$266° \pm 5°$
(iii)	1960–1968	420 ± 20	$270° \pm 5°$
Thomas, assuming the beacon to be stationary.	1966–1967	349 ± 2	$272° \pm 2°$

In Table 3 the ice velocity at the Halley Bay station for 1966–1967 is compared with independent estimates obtained from:

(i) Surveys of positions on the ice shelf with respect to prominent grounded icebergs. This was attempted twice (Limbert, 1964; Ardus, 1965) and in each case, of the several icebergs observed, only one could reliably be used as a control point for both the sets of observations necessary. Consequently the quoted errors accommodate neither possible iceberg movement nor weathering of the feature observed. Furthermore the iceberg chosen by Ardus as fixed was probably one which Limbert had earlier suspected of movement (Limbert, personal communication), and for this reason his results are not included.

(ii) Comparison of astro-fixes made in 1956–1957 with those of 1966–1967.

(iii) Repeated mapping of positions on the ice shelf with respect to a north–south oriented magnetic anomaly (Jamieson, personal communication).

The mean of these independent estimates is 50 m yr^{-1} greater than the 1966–1967 value. Annual variations in velocity are unlikely to be sufficiently large to account for this difference, which is probably due to movement of the grounded ice beneath the beacon.

In Fig. 7 the movement vectors at each stake have been adjusted for an assumed ice velocity at the beacon of 50 m yr^{-1} to the west. Before this correction was applied the movement vectors on the inland ice sheet were inclined at 40° to 50° to the formlines, whereas in Fig. 7 they are seen to be perpendicular, which is more in accordance with predictions from theory (Nye, 1965, p. 517) and is a further indication that the velocity assigned to the beacon is approximately correct.

6. Conclusions

The method described for computing positions from survey data obtained on a surface of moving ice produces results with an accuracy comparable with that of the data, despite appreciable deformation throughout the period of the survey.

Additional errors due to curvature and acceleration of the flow lines were detected only in the observations made near the McDonald Ice Rumples and across the junction between floating and grounded ice, where epoch positions based on linear interpolation are in error. Because the two sets of observations were separated by only one year this error remains within tolerable limits. For surveys separated by several years a more sophisticated approach may be necessary. For example:

(i) Using two sets of observations, compute positions and ice velocities as described above, and then recompute all positions using, to improve the interpolated epoch positions, the values for the curvature and acceleration of the flow lines obtained from the first computation.

(ii) Taking three sets of observations and computing three positions for each stake. Interpolation of epoch positions can then be made by assuming, for instance, that the curvature and acceleration of each flow-line are constants.

Using the computed stake positions it was possible to transfer all astro-fixes to their equivalent values at the beacon. Those observed near and on the inland ice sheet gave positions which share a consistent difference of 15 ± 3 seconds of latitude and 40 ± 5 seconds of longitude when compared with those observed at the northern end of the stake network.

Although anomalous refraction conditions might account for a small part of this difference, the most likely explanation is deviation of the vertical towards the continent at the southern end of the stake line. The necessary 15 seconds deviation between positions only 60 km apart implies an intense local gravity anomaly, and a gravimetric survey of the area may well yield interesting results.

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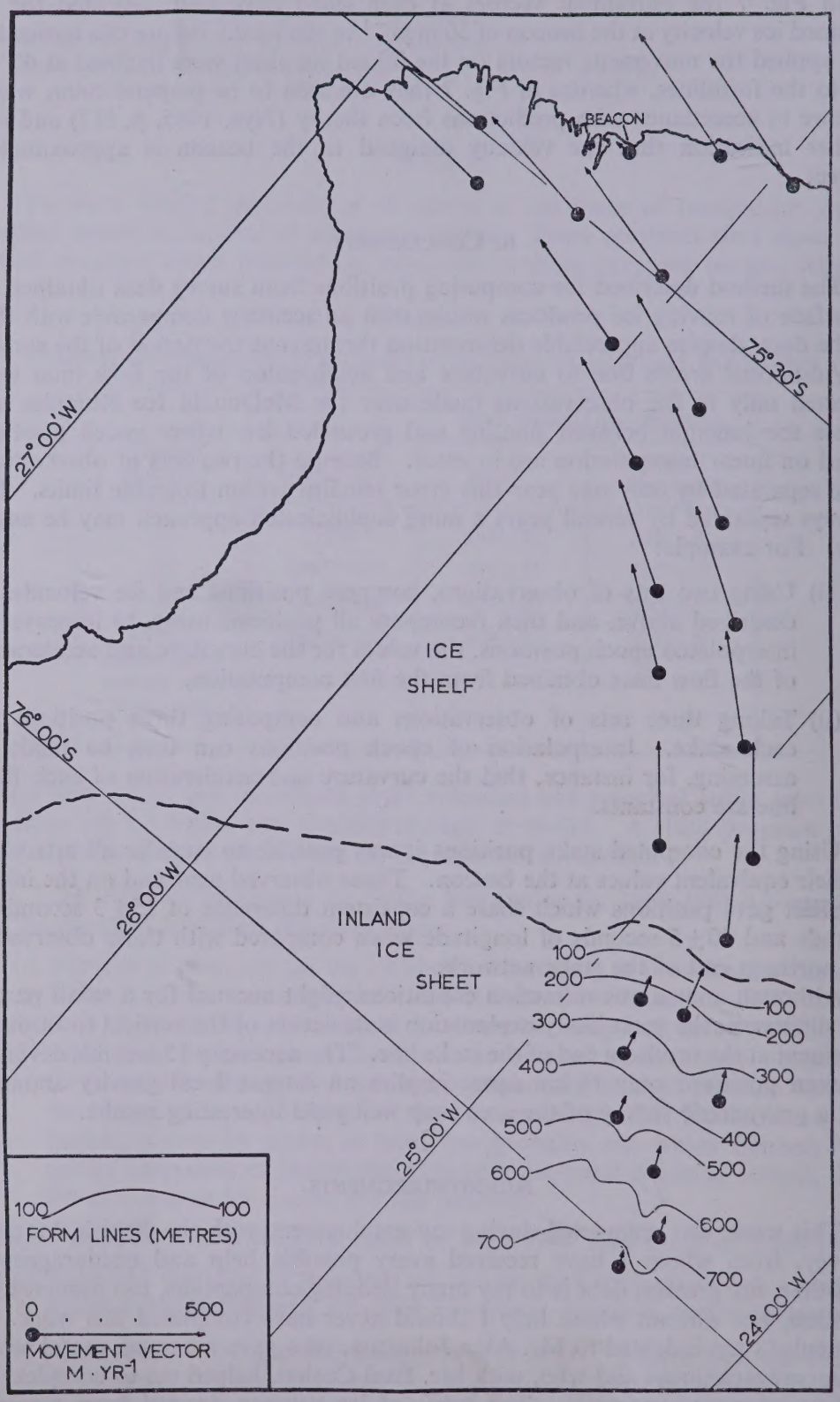

Fig. 7. Movement vectors at a selection of the markers.

Mr. James Jamieson, who also made the 1968 magnetic survey. Miss L. M. Windsor of the Directorate of Overseas Surveys brought to my notice the mid-latitude formulae of H. F. Rainsford, and supplied the values of azimuth and distance obtained using these formulae. Mr. B. M. E. Smith introduced me to computers and patiently assisted me through many programming difficulties. Finally I wish to thank Dr. Egon Dorrer, Mr. J. E. Jackson and Dr. C. W. M. Swithinbank for carefully reading and helpfully criticising the manuscript.

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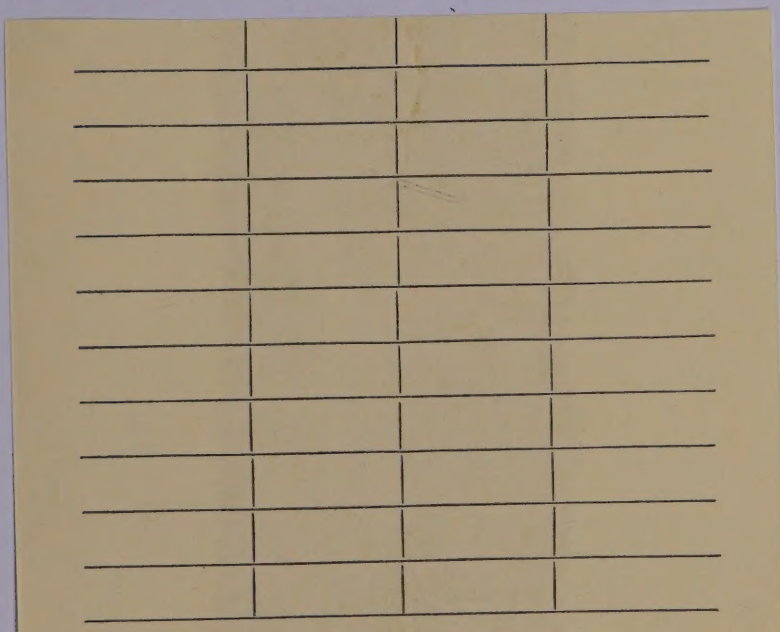

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